Transmission technical calculations – Main Formulas

Size designations and units according to the SI-units

Linear movement:

$v = \frac{s}{t}$	m/s
$s = v \times t$	m
$s_a = \frac{1}{2} \times a \times t_a^{\ 2}$	m
$a = \frac{v}{t_a}$	m/s^2
$P = F \times v$	W
$F = m \times a$	N
$W = F \times s$	Ws
$W = \frac{m \times v^2}{2}$	Ws

Rotation

$\omega = 2 \times \pi \times f$	rad/s
$v = \omega \times r = 2 \times \pi \times f \times r$	m/s
$M = F \times r$	Nm
$P = M \times \omega$	W
$M = J \times \dot{\omega}$	Nm
$W = \frac{J \times \omega^2}{2}$	Ws or J
$J = m \times r^2$	kgm^2

Units used

v = velocity in m/s

M = mass in kg

P= power in W

s = length in m

t = time in sec.

a = acc. in m/s^2

t_a = acc. time in sec.

F= force in N

W = work in Ws = J = Nm

ω = ang. velocity in rad./sec.

f = frequency in rev./sec.

r = radius in m

M = torque in Nm

J = Rotational mass moment of inertia in kgm^2

$\dot{\omega}$ = angular acc. in rad/s^2

M_a= acceleration torque in Nm

Formulae for the transmission technique

Power

Rotational Movement:

$$P_s = M \times \omega$$ W (without loss)

$$\omega = \frac{\pi \times n}{30}$$ rad/s

$$P = \frac{M \times n}{\eta} \times \frac{\pi}{30}$$ W

$$P = \frac{M \times n}{\eta} \times \frac{\pi}{30} \times \frac{1}{1000}$$ kW

Torque

$$M = F \times r$$ Nm

$$M_A = \frac{P \times 9550}{n} \times \eta$$ Nm

Linear Movement:

$$P = F \times v \times \frac{1}{\eta}$$ W

$$P = \frac{F \times v}{1000} \times \frac{1}{\eta}$$ kW

Lead screw:

$$M = \frac{F \times p}{2000 \times \pi \times \eta}$$ Nm

Toothed belt:

$$v = \pi \times D \times n$$ m/min

$$m = \frac{D}{z}$$

$$D = \frac{z \times t}{\pi}$$

Units used

M = torque in Nm
ω = angular velocity in rad/s
n = revolutions/min.
η = efficiency (motor)
F = force in N
z = number of teeth
t = distance between teeth in mm

v = velocity in m/min
M_A = delivered torque in Nm
r = radius in m
P= power in kW or W
D= diameter in m
m = module
p = pitch in mm/rev
P_s = transmitted shaft power
P= necessary motor power

Acceleration torque

$$M_a = \frac{J \times n}{t_a} \times \frac{\pi}{30}$$ Nm

For operation of electrical motors with gear transmission:

$$M_a = \frac{J_{red} \times n}{t_a} \times \frac{\pi}{30}$$ Nm

Reduction of rotational mass moment of inertia

$$J_{red} = \frac{n^2}{n^2_{mot}} \times J = \frac{1}{i^2} \times J$$ kgm^2

Linearly moveable masses is reduced to the number of revolutions of the motor according to

$$J_{red} = 91.2 \times m \times \frac{v^2}{n^2_{mot}}$$ kgm^2

Rotational mass moment of inertia of a solid cylinder

$$J = \frac{1}{2} \times m \times r^2_y$$ kgm^2

Units used

M_a = acceleration torque in Nm

J = rotational mass moment of inertia in kgm^2

n = number of revolutions in rev./min.

t_a = acceleration time in s

v = velocity in m/s

$i = \frac{n_{mot}}{n}$ gear ratio

J_{red} = reduced rotational mass moment referred to the motor shaft in kgm^2

n_{mot} = number of revolutions of motor in rev/min.

m = mass in kg

r_y = outer radius of solid cylinder

Acceleration and deceleration time

$$t_a = \frac{J \times n}{9.55 \times M_a}$$ s

Braking work

$$A = \frac{M_b}{M_b + M_L} \times \frac{J_{red} \times n^2{}_{mot}}{182.4}$$ Ws

Necessary power for linear movement

$$P = \frac{F \times v}{1000 \times \eta}$$ kW

Force at sliding friction

$$F = m \times g \times \mu$$ N

Units used

M_a = acceleration torque in Nm

M_b = braking torque in Nm

M_L = load torque reduced to the motorshaft in Nm

t_a = acceleration time in s

J = rotational mass moment of inertia in kgm^2

n = Number of revolutions in rev./min.

W = work in Ws or J

η = efficiency of linear movement

μ = friction coefficient

J_{red} = reduced rotational mass moment of inertia referred to the motor shaft

n_{mot} = number of revolutions of motor in rev/min.

P = power in kW

F = force in N

v = linear velocity in m/s

m = load in kg

g = gravity (9.81 m/s^2)

Frictional force during linear movement using wheels or rails

$$F = \frac{2 \times m \times g}{D} \times (\mu_1 \times \frac{d}{2} + f) \times \mu_2 \quad \text{N}$$

By approximate calculations it is often simple to use the specific running resistance R in N/ton carriage weight by calculation of the required power.

$$P = \frac{R \times q \times v}{1000 \times \eta} \quad \text{kW}$$

Heavier carriages on rails, roller bearings R = 70 – 100N/ton

Lighter carriages on rails, roller bearings R = 100 – 150N/ton

Units used:

F = force in N

m = load in kg

g = gravity

D = wheel- or roller diameter in m

f = rolling friction radius

d = shaft diameter in m

μ_1 = bearing friction

μ_2 = rail- or side friction

v = velocity in m/s

η = efficiency

q = load in ton

Rolling friction radius, f (m):

Steel against steel	f = 0.0003 – 0.0008
Steel against wood	f = 0.0012
Hard rubber against steel	f = 0.007 – 0.02
Hard rubber against concrete	f = 0.01 – 0.02
Inflated rubber tire against concrete	f = 0.004 – 0.025

Bearing, rail- and side friction:

Roller bearings μ_1 = 0.005
Sliding bearings μ_1 = 0.08 – 0.1
Roller bearings μ_2 = 1.6
Slide bearings μ_2 = 1.15
Sideguides with rollerbearings μ_2 = 1.1
Roller guides side friction μ_2 = 1.8

SI - Units

SI basic units

Symbol	Measure	Unit
m	length	metre
kg	mass	kilogram
s	time	second
A	electrical current	ampere
K	temperature	Kelvin

For Motion Control

Designation	Measure	Unit	Symbol
a	distance	metre	m
α,β	angle	radian	rad
	angle	degree	°
d	diameter	metre	m
h	height	metre	m
l	length	metre	m
r	radius	metre	m
s	distance	metre	m
V	volume	cubic-metre	m^3
a	linear acceleration		m/s^2
$\dot{\omega}$	angular acc.		rad/s^2
f	frequency	Hertz	Hz
g	gravity		m/s^2
n	revolutions per unit	rev./min.	1/s
w	angular velocity		rad/s
T	time constant	second	s
t	time	second	s
v	linear velocity		m/s

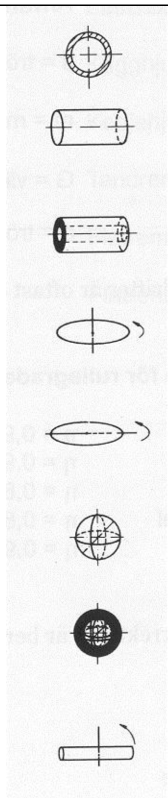
Mechanical

Designation	Measure	Unit	Symbol
F	force	Newton	N
G	weight force	Newton	N
J	Rotational mass moment of inertia		kgm^2
M	torque	Newtonmetre	Nm
m	mass	kilogram	kg
P	power	Watt	W
W	energy	Joule	J
η	efficiency		
μ	friction coefficient		
i	gear ratio		

Electrical

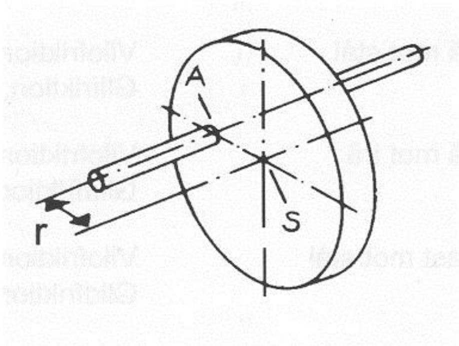
Designation	Measure	Unit	Symbol
I	current	Ampere	A
P	active power	Watt	W
R	resistance	Ohm	Ω
S,Ps	appearent power	Voltampere	W, VA
U	voltage	Volt	V

The rotating mass moment of inertia of rotating bodies

Body	Rotation	Symbol	Rotational mass moment of inertia, J in kgm^2
Hollow cylinder	Around own axis		$m \times r^2$
Homogeneous cylinder	Around own axis		$\frac{m}{2} \times r^2$
Thickwalled cylinder	Around own axis		$\frac{m}{2} \times (r^2{}_1 + r^2{}_2)$
Disc	Around own axis		$\frac{m}{2} \times r^2$
Disc	Around own plane		$\frac{m}{4} \times r^2$
Sphere	Around own center		$\frac{2 \times m}{5} \times r^2$
Thinwalled sphere	Around own center		$\frac{2 \times m}{3} \times r^2$
Thin rod	Perpendicular around own axis		$\frac{m}{12} \times l^2$

Steiners Equation

Rotational mass moment of inertia relative to a parallel shaft in the distance a



$J = J_0 + m \times r^2$	kgm^2
J_0 = rotational mass moment of inertia relative to the center of gravity axis	kgm^2
m = mass of the body	kg
r = shaft distance	m

Correlation between rotational mass moment of inertia and rotating mass

$J = m \times r_J^{\,2}$ kgm^2

Units used:

J = rotational mass moment of inertia in kgm^2

m = mass in kg

r_J = inertial radius in m

The efficiency for different types of drives are often values obtained by experience:

Some normal values for parts with rollerbearings:

Drive belt with 180° force transmitting angle	η = 0.9 – 0.95
Chain with 180° force transmitting angle	η = 0.9 – 0.96
Toothed rod	η = 0.8 – 0.9
Transporting belt with 180° transmitting angle	η = 0.8 – 0.85
Wire with 180° transmitting angle	η = 0.9 – 0.95

The friction values are difficult to give correctly and are dependant on surface conditions and lubrication.

Some normal values:

Steel against steel	static friction, dry	μ = 0.12 – 0.6
	dynamic friction, dry	μ = 0.08 – 0.5
	static friction, viscous	μ = 0.12 – 0.35
	dynamic friction, viscous	μ = 0.04 – 0.25
Wood against steel	static friction, dry	μ = 0.45 – 0.75
	Dynamic friction, viscous	μ = 0.3 – 0.6
Wood against wood	Static friction, dry	μ = 0.4 – 0.75
	Dynamic friction, viscous	μ = 0.3 – 0.5
Plastic against steel	static friction, dry	μ = 0.2 – 0.45
	Dynamic friction, viscous	μ = 0.18 – 0.35

The twisting torque on a shaft from a pulling force is according to the following

$$M = F \times r \times \eta = F \times \frac{d_0}{2} \times \eta \qquad \text{Nm}$$

Units used

F = cross force

M = twisting torque

d_o = effective diameter on toothed wheel or chain wheel

η = efficiency

r = radius in m

Efficiency, η

Toothed wheel	= 0.95
Chain wheel	= 0.95
Toothed belt	= 0.80
Flat belt	= 0.40
Flat belt, pre-tensed	= 0.20